PedalScope Methods

Technical reference: how each measurement is made, and how to check it

PedalScope

Abstract

This document states, for each measurement PedalScope makes, exactly what the shipping code computes: the stimulus, the analysis pipeline step by step with its equations, every parameter that determines the result, and the limits of what the result can claim. Every number in it is generated from a machine-readable methods manifest emitted by the shipped code (the source is open: `https://github.com/PhaseDog/PedalScope`), every method is reimplemented independently in Python from that manifest and the published definition, and a continuous-integration parity test test holds the two implementations to each other and to analytic ground truth. A method change in the app that the Python does not follow is a failing build, not a stale page.

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Contents

1 How to read this document, and how to trust it

1.1 The pipeline behind the pages

1. **The manifest.** `analysisdump manifest` (a package tool built on the app's own measurement kernel) emits `Docs/Methods/generated/manifest.json`: one object per measurement, every parameter obtained by calling the shipped function that owns it, never by transcription. Where a parameter is a function rather than a number — a window shape, a gate — the manifest records its name, its defining constants and a one-line rule.

2. **The reimplementation.** `Docs/Methods/py/harmonic_distortion.py` is the method written again, from the manifest and the published definition [1, 3], not from the Swift; each further chapter has its own (`transfer_curve.py` for the Transfer Curve). It produces the same table the shipped tool produces.

3. **The parity test.** `test_parity_hd.py` (and, per chapter, its sibling) runs the shipped code and the Python on identical synthetic devices and asserts, at every valid point, that they agree to a stated tolerance and that both agree with analytic truth to a stated tolerance. Tolerances were set from measured distributions and are quoted in §3.7 and §4.7.

4. **The figures** are generated by the Python, seeded and byte-stable; the UI captures are rendered headlessly from the app's own chart views by the user-guide pipeline.

5. **The drift check.** CI regenerates the manifest, the fragments and the figures and fails on any difference from the committed copies, so what this document states is what the build computed.

1.2 What this document does not do

It does not describe what any measurement sounds like. A measured difference is a measured difference; whether it can be heard is a question the instrument does not answer, and no sentence here claims otherwise. It names no third-party product in a finding.

2 Notation and conventions

Symbol	Meaning
f_s	the sample rate, Hz
f_1, f_2	the sweep's start and end frequencies, Hz
L	the sweep's rate constant, s: $f(t) = f_1 e^{t/L}$
T	the sweep's actual duration, $T = L \ln(f_2/f_1)$
A	the sweep amplitude, linear, 1 = digital full scale
$\varphi(t)$	the sweep's instantaneous phase, rad
$f(t)$	the sweep's instantaneous frequency, Hz
k	a harmonic order; $k = 1$ is the fundamental
K	the highest order extracted
f_0	a fundamental frequency, Hz — the excitation whose k-th harmonic is read
$H_k(f)$	the harmonic response of order k on the output-frequency axis (§5)
Δt_k	the time advance of order k's harmonic pulse: $L \ln k$
N	the deconvolution DFT length; M the per-order response DFT length
T_k	the extraction window duration for order k

Symbol	Meaning
$h[n]$	the deconvolved impulse response, circular over N samples
dB re input	$20\log_{10}|H_k|$ — input-normalized: a unit-gain linear device reads $H_1 = 1$, i.e. 0 dB
dBFS	dB relative to digital full scale
dBc	dB relative to a carrier (here, the louder played tone or the fundamental)

Angles are radians. Logarithms are natural unless written $\log_{10}$. Time-domain indices are integers $n = 0 \dots N-1$; "circular" means modulo the buffer length. A raised cosine is $\frac{1}{2}(1-\cos\theta)$ rising or $\frac{1}{2}(1+\cos\theta)$ falling.

3 Harmonic Distortion

3.1 Purpose, and what the measurement cannot tell you

Harmonic Distortion characterises how a device converts a single input frequency into output at integer multiples of it: for every fundamental f_0 in the swept band and every order $k \le K$, the amplitude the device emits at kf_0, normalized by the input amplitude. One synchronized swept sine captures the whole band at one drive level; the result is K frequency responses $H_1(f) \dots H_K(f)$ and the derived total harmonic distortion curve.

What it cannot tell you, stated once: (a) anything about behaviour at other drive levels — one sweep is one operating point; the Gain Map and Compression measurements own the level axis; (b) anything about intermodulation between two simultaneous notes — Chord IMD owns that; (c) whether a read in an order is content the circuit shaped or noise that landed in that order's analysis window — the harmonic shaping predicate reads that separately (§3.4); (d) anything below the rig's own measurement floor (§3.4); and (e) anything about what the distortion sounds like.

3.2 The stimulus

The stimulus is a synchronized exponential sine sweep [2, 3]:

$$x[n] = A \sin\varphi(t_n), \qquad \varphi(t) = 2\pi f_1 L\left(e^{t/L} - 1\right), \qquad t_n = n/f_s,\ n = 0 \dots \lfloor Tf_s + \tfrac{1}{2}\rfloor - 1. \quad (1)$$

Its instantaneous frequency is $f(t) = f_1 e^{t/L}$, so the sweep reaches f_2 at $T = L\ln(f_2/f_1)$. The rate constant is not free: the synchronization condition requires $f_1 L$ to be an integer, so for a requested duration $\hat{T}$

$$L = \frac{\max(1,\ \mathrm{round}(f_1\hat{T}/\ln(f_2/f_1)))}{f_1}, \quad (2)$$

which is what makes the k-th harmonic generated by a memoryless nonlinearity an exact time-advanced copy of the sweep, advanced by

$$\Delta t_k = L\ln k, \quad (3)$$

and therefore separable in the deconvolved response with meaningful phase. The first $\lfloor t_{\mathrm{in}} f_s\rfloor$ samples are scaled by a rising raised cosine and the last $\lfloor t_{\mathrm{out}} f_s\rfloor$ by a falling one. The fades bound the excited band: below $f(t_{\mathrm{in}})$ and above $f(T - t_{\mathrm{out}})$ the device was driven under a ramp, and reads there are properties of the measurement, not the device.

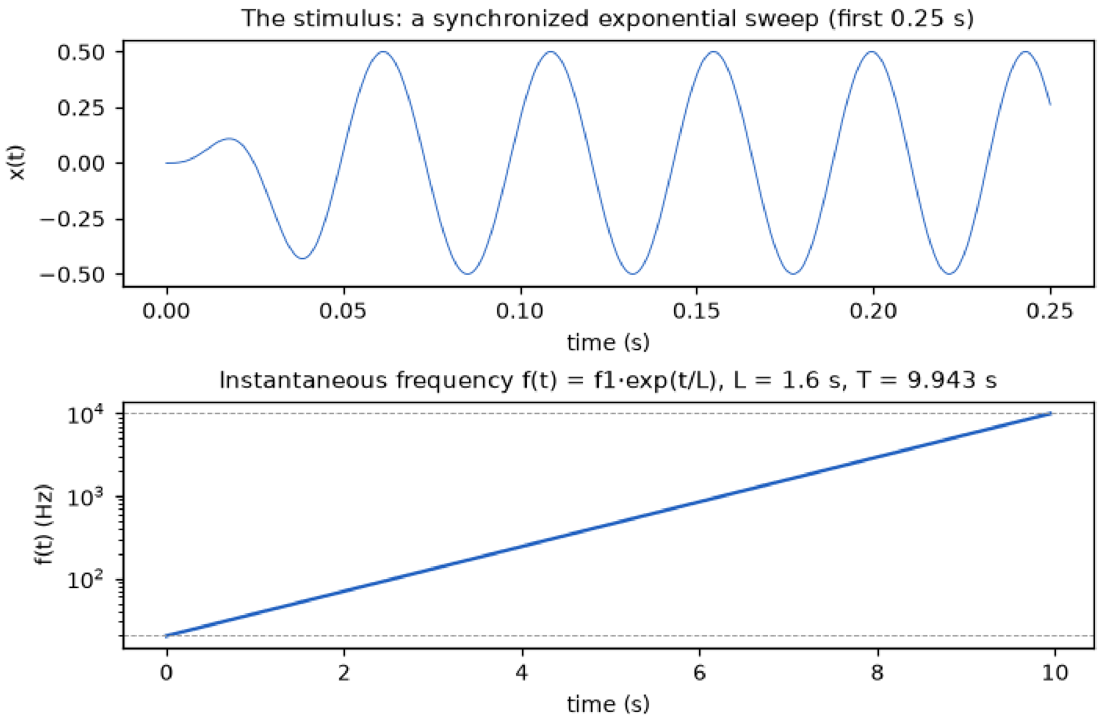

Figure 1: The stimulus, from the reimplementation: the waveform's first quarter second and the instantaneous frequency over the full sweep.

3.3 The pipeline

Given the captured response $y[n]$, time-aligned with the sweep's start (the app measures and removes the loop latency per capture before analysis; the oracle runs at zero latency):

1. **deconvolution with the analytic inverse.** Zero-pad y to N, the smallest power of two not less than the capture length plus $\lfloor 0.1 f_s \rfloor$, and take its DFT $Y[m]$. The inverse filter is the stationary-phase closed form [3]

$$\tilde{X}^{-1}(f) = 2\sqrt{f/L}\, \exp\Big(j\big(\tfrac{\pi}{4} - 2\pi f L\,(1 - \ln(f/f_1))\big)\Big), \tag{4}$$

evaluated at $f = mf_s/N$ for $0 < m < N/2$ and mirrored to the negative bins, tapered to zero below f_1 (a rising raised cosine from $f_1/2$ to f_1) with the DC and Nyquist bins zero. Using the closed form rather than inverting the sweep's own spectrum extrapolates the phase law above f_2, which is what lets harmonics of high fundamentals — whose output frequencies exceed f_2 — deconvolve into clean pulses all the way to Nyquist. The impulse response is

$$h[n] = \frac{1}{N f_s}\, \mathrm{IDFT}\{Y[m]\, \tilde{X}^{-1}(mf_s/N)\}[n], \tag{5}$$

the $1/N$ being the inverse DFT's scaling and the $1/f_s$ converting a sampled signal's DFT to the continuous-time spectrum the analytic inverse expects. A unit-gain linear device yields a pulse of height A at $n = 0$.

2. **Harmonic separation.** Order k's pulse arrives Δt_k earlier than the linear one, i.e. at circular index $N - \mathrm{round}(\Delta t_k f_s)$ (Figure 2).

3. **Windowing.** Each order is cut out with a raised-cosine window of half-widths ℓ_k (earlier

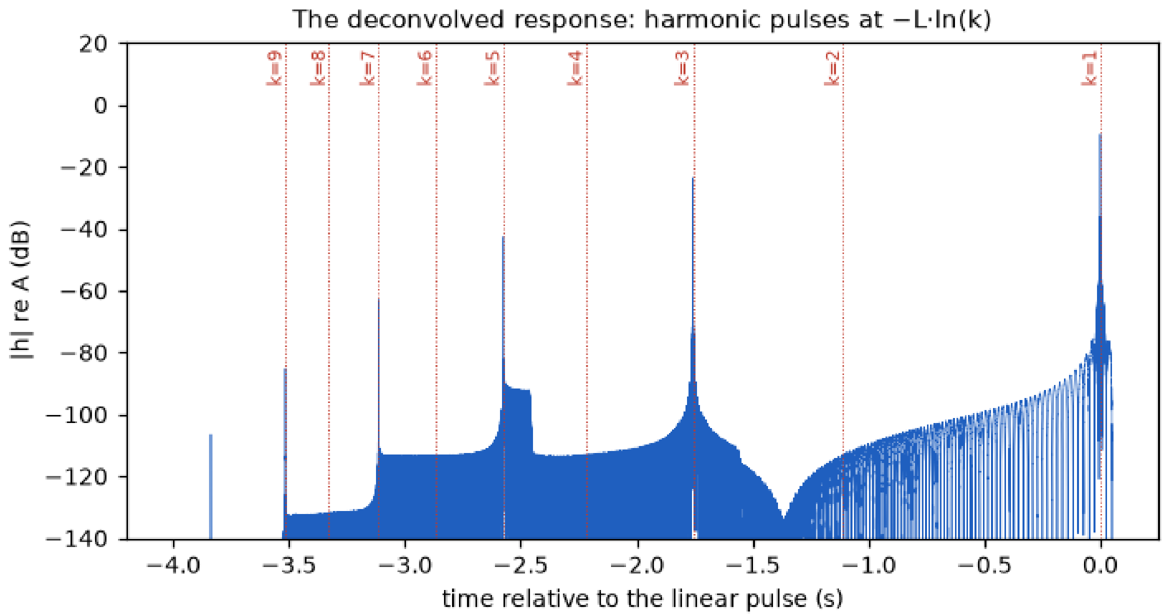

Figure 2: The deconvolved response of the worked example (a symmetric soft clipper), on a negative-delay axis. Odd orders carry content; the even orders' pulses are absent, as they must be for a symmetric device.

in time, toward order $k+1$) and r_k (toward order $k-1$), in samples:

$$c = \min\left(\tfrac{M}{2} - 1,\ t_{\max} f_s\right), \qquad \ell_k = \left\lfloor \min\left(c,\ \tfrac{(\Delta t_{k+1} - \Delta t_k) f_s}{2}\right) \right\rfloor,$$
$$r_k = \begin{cases} \lfloor c \rfloor & k = 1 \\ \left\lfloor \min\left(c,\ \tfrac{(\Delta t_k - \Delta t_{k-1}) f_s}{2}\right) \right\rfloor & k > 1, \end{cases} \tag{6}$$

with $t_{\max}$ the half-width cap. The window is $w(o) = \frac{1}{2}(1 + \cos(\pi |o| / \ell_k))$ for $-\ell_k \le o < 0$ and $\frac{1}{2}(1 + \cos(\pi o / r_k))$ for $0 \le o < r_k$: unity at the pulse, zero at each edge. An order is emitted only while $\ell_k \ge 16$ and $r_k \ge 16$; the analyzer stops at the first order that fails, so a short sweep simply carries fewer orders. The window duration $T_k = (\ell_k + r_k)/f_s$ is what sets order k's spectral resolution and the correlation width the shaping predicate must clear.

4. **Spectra.** The windowed segment is placed with the pulse at circular index 0 of an M-point buffer (so no linear-phase correction is needed) and transformed:

$$H_k[b] = \frac{1}{A} \mathrm{DFT}_M\{\, h[(p_k + o) \bmod N]\, w(o) \,\}[b], \qquad b = 0 \ldots M/2, \tag{7}$$

with p_k the pulse position. The $1/A$ is the input-normalized convention: $H_1 = 1$ for a unit-gain linear device.

5. **Reading.** H_k lives on the *output* frequency axis. The k-th harmonic of a fundamental f_0 is read at $f = k f_0$, $|H_k|$ interpolated linearly between the two bins adjacent to $f/(f_s/M)$, and reported as $20 \log_{10} |H_k(k f_0)|$.

6. **Validity.** A read is trustworthy only inside the validity bound: $f_1 \le f_0 \le f_2$ and

$$f_0 \le \frac{f_s}{2k+1}, \tag{8}$$

beyond which the capture folds order $k+1$ onto the read at $k f_0$ (the alias image bound), and on a loopback-compensated record additionally $f_0 \le f_{\mathrm{cal}}/k$ with f_{cal} the calibration

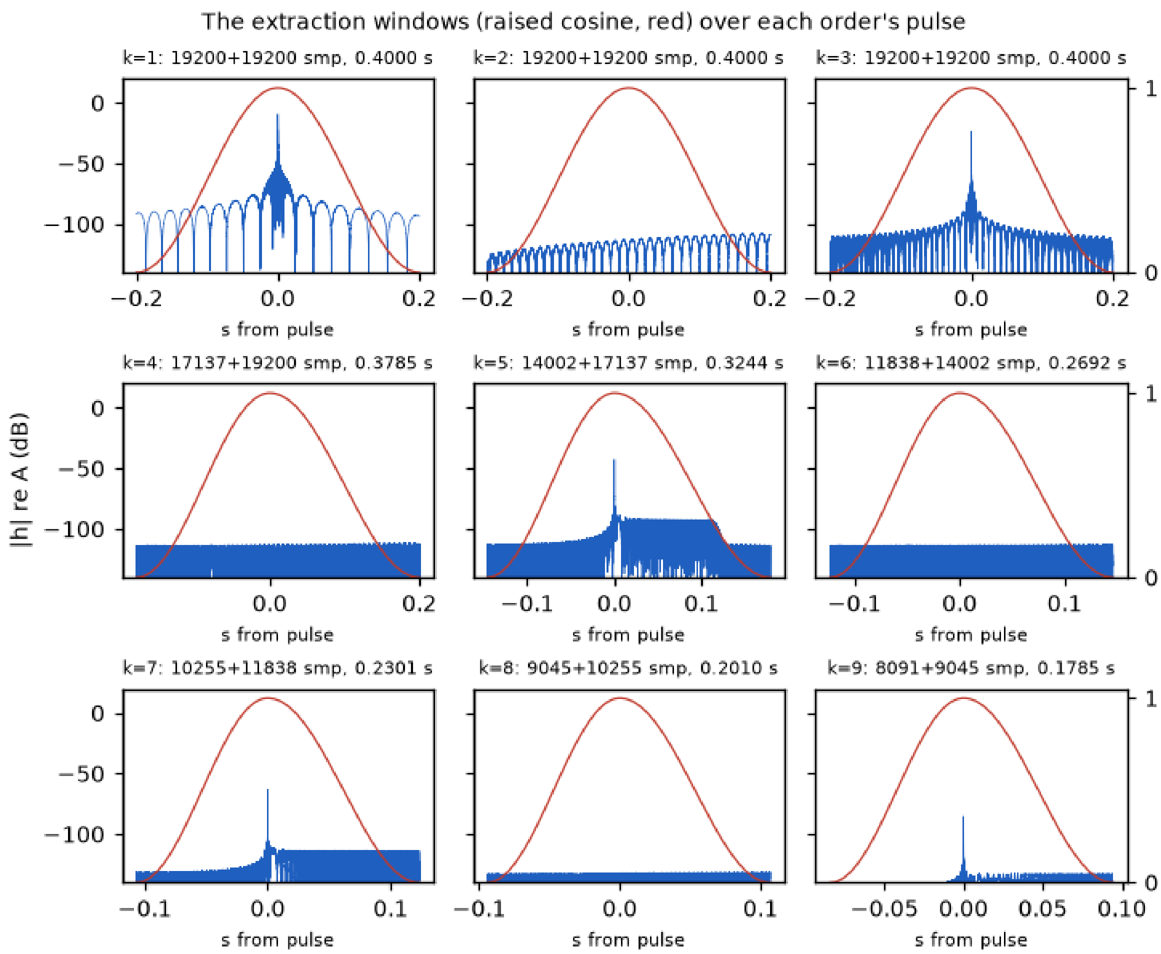

Figure 3: The nine extraction windows over their pulses. Orders 1–3 take the full cap; from order 4 the gap to the neighbouring pulse binds.

sweep's excited-band top. Invalid reads are excluded from every fit, feature, summary and comparison.

7. **THD.** With the audible band B,

$$\mathrm{THD}(f_0) = \frac{\sqrt{\sum_{k\geq 2,\ kf_0\in B,\ \text{valid}} |H_k(kf_0)|^2}}{|H_1(f_0)|}, \tag{9}$$

THD and not THD+N; a point where the validity bound removed an in-band order is flagged truncated. The curve is sampled on a log-spaced grid of fundamentals from $1.5f_1$ (the first fraction of the sweep is fade-in territory) to f_2.

3.4 Read-time predicates on the result

Three further judgements are computed from the stored spectra when a record is displayed or compared; none is persisted, so fixing a predicate fixes every stored record.

The floor. The rig's own floor comes from the embedded calibration's pooled THD-vs- frequency stamp (one loopback sweep), read at f_0 by linear interpolation on the $(\log f,\ \mathrm{dB})$ plane, then adjusted: per track by $-10\log_{10} N_{\text{tracks}}$ (the pooled figure spread over the harmonic tracks), for drive by the calibration sweep level minus the record's drive, and for sweep averaging by $-10\log_{10}(N/N_{\text{cal}})$ — applied only where the residual the stamp measures is noise (inside the validity bound, above the fade-in edge). A plugin record's operative floor enters as a second, never-averaged bound and the applicable floor is the larger. A read at or below its floor is a bound, not a figure.

Fundamental presence. Every ratio to H_1 (THD above all) is arithmetic unless the fundamental is a measurement: a fundamental sitting at or beyond the analyzer's separation bound below the loudest valid, in-band order is fundamental presence-absent and nothing is divided by it.

Shaping. Anything inside order k's window is reported as H_k, the device's own hiss included. Whether a read is content the circuit deterministically shaped is decided by the coherence of the complex response along frequency: with a lag of $\lceil \gamma/(T_k \cdot f_s/M) \rceil$ bins (clear of the window's own spectral correlation) and n samples,

$$z_i = H(b_i + \mathrm{lag})\,\overline{H(b_i)}, \qquad \Gamma = \frac{|\sum_i z_i|}{\sum_i |z_i|}, \tag{10}$$

which for pure noise is $1/\sqrt{n}$. Γ maps to a signal-to-noise ratio through a pinned Monte-Carlo table, that ratio to the read's own 1-σ dB read scatter (the Rician mean-of-dB uncertainty [4]), and a read is shaped when its ratio clears the bar. The predicate's constants and both tables are in the parameter table below.

Averaging. N back-to-back sweeps in one capture are analyzed separately, each repeat derotated against the first by one fitted delay for all orders (the delay δ maximizing $\mathrm{Re}\sum z\, e^{j\omega\delta}$ over cross-spectral terms sampled per order), and the complex spectra are averaged bin by bin. Magnitudes are never averaged: N reads of a biased quantity average to the same biased number more precisely.

3.5 Parameters

Every value below is read from the manifest, which read it from the shipped code. The per-order table follows.

Parameter	Value
Stimulus	
start frequency f_1	20 Hz
end frequency f_2	10 000 Hz
requested duration $\hat{T}$	10 s
sample rate f_s	96 000 Hz
amplitude A (re full scale)	0.5
rate constant L	1.6 s
synchronization cycles $f_1 L$	32
actual duration $T = L \ln(f_2/f_1)$	9.943372957 s
sample count	954 564
fade-in / fade-out	0.05 s / 0.01 s
excited band (fade edges)	20.635 Hz – 9937.7 Hz
Deconvolution	
inverse-filter taper	10 Hz – 20 Hz
DFT length N for one synth capture	1 048 576
Extraction	
harmonic count K	9
response DFT length M	65 536
window half-width cap	0.2 s
minimum half-width	16 samples (a literal in the analyzer)
bin width f_s/M	1.4648438 Hz
bins per response	32 769
Grids	
distance grid (§11)	96 log-spaced points, 30–10 000 Hz
THD curve grid	127 of 128 requested, 30–9552.89 Hz
oracle grid (`synth`)	48 points, 30–10 000 Hz
distance fixed floor clamp	-80 dB
distance minimum resolved share	0.2
audible band	20 Hz – 20 000 Hz
Floor	
calibration analyzed sweeps N_{cal}	1
averaging term at 1 / 2 / 4 / 8 sweeps	0 dB / -3.01 dB / -6.021 dB / -9.031 dB
stamp read (median points)	1
at-floor epsilon	0.1 dB
Averaging	
default / offered sweep counts	4 / 1, 2, 4, 8
pre-roll / gap / tail	0.5 s / 0.5 s / 1 s
delay fit: samples per order	48
delay fit: scan limit / coarse / fine	±8 / 0.02 / 0.0005 samples
Fundamental presence	
analyzer separation bound	76 dB
verdict grid points	128
Shaping	
window samples	32
pure-noise coherence $1/\sqrt{n}$	0.17678
decorrelation factor	2.4

Parameter	Value
shaped SNR bar	6 dB
unshaped scatter	5.57 dB
Summaries	
lexicon version	1.2.2
Oracle (`analysisdump synth`)	
post-sweep pad	48 000 samples
phantom Nyquist gate	$0.98 \times f_s/2$
ground-truth quadrature points	262 144
noise generator increment	`0x9E3779B97F4A7C15`

k	$\Delta t_k = L \ln k$ (s)	window T_k (s)	window (samples)	max f_0 valid (Hz)
1	0	0.4	38 400	10 000
2	1.10904	0.4	38 400	10 000
3	1.75778	0.4	38 400	10 000
4	2.21807	0.37851	36 337	10 000
5	2.5751	0.324365	31 139	8727.27
6	2.86682	0.269167	25 840	7384.62
7	3.11346	0.230135	22 093	6 400
8	3.32711	0.201042	19 300	5647.06
9	3.51556	0.1785	17 136	5052.63

3.6 The result

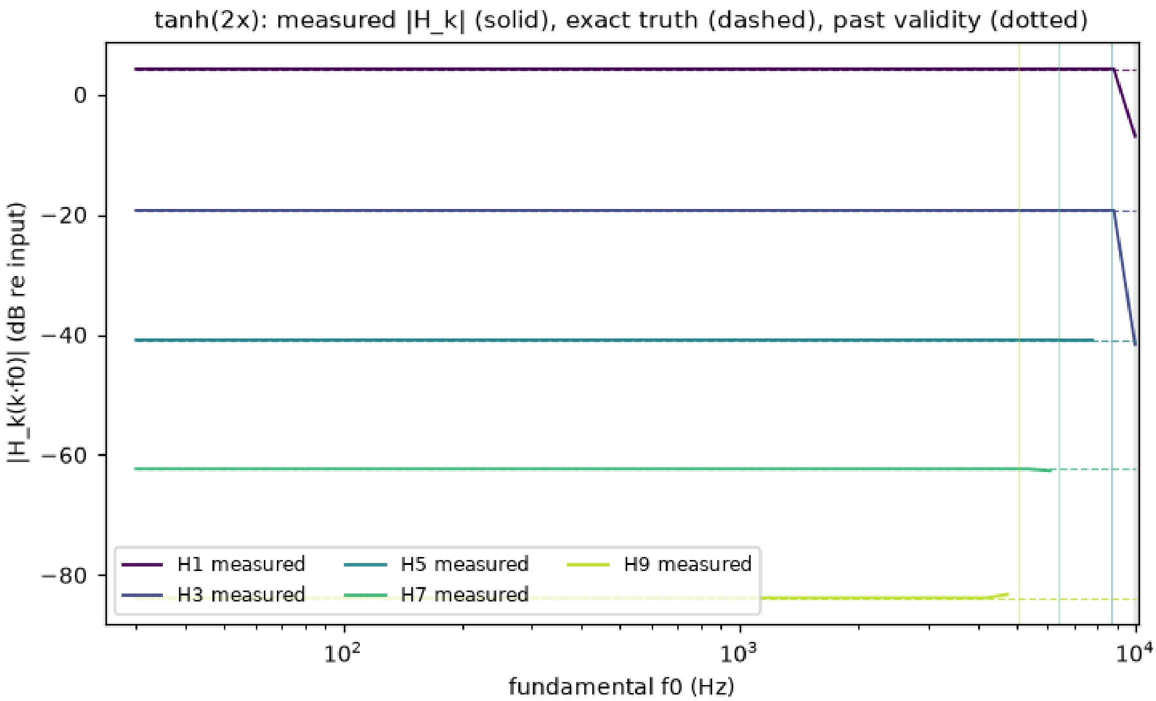

Figure 4: The worked example, $y = \tanh(2x)$ at $A = 0.5$: measured $|H_k|$ for the odd orders against the exact Fourier amplitude of the nonlinearity at the sweep amplitude. Dotted segments lie past each order's validity bound; the shaded strip past the fade-out edge is the excited band's end, where the last grid point reads the ramp, not the device.

The app's own rendering of the same measurement kind, from the user guide's figure pipeline (a simulated device, rendered headlessly from the chart views, never screenshotted):

Figure 5: The Harmonic Distortion chart and the Distortion-vs-Note chart as the app draws them, from the guide's simulated fixture.

3.7 Worked example and parity

The parity test runs four synthetic devices through both implementations at one seed with no noise: a phantom capture of three orders (exact by construction), $\tanh(2x)$, a hard clip at ± 0.1 under an amplitude of 0.5, and $x + 0.2x^2 + 0.3x^3$. For every valid point inside the excited band it asserts:

- $|\text{Python} - \text{Swift}| \leq 1e-08\,\text{dB}$ on every order that carries content (measured maximum 2.1e-11 dB on this run; the two implementations differ only by DFT and transcendental rounding);
- both against analytic truth: $\leq 0.01\,\text{dB}$ over the whole band for a phantom, and over the lower half of each order's validity band for the per-sample nonlinearities (tanh 0.01 dB, poly 0.01 dB, hardclip 0.3 dB);
- analytically empty orders read at least 20 dB below the case's loudest order on both sides, and the two sides agree on that residue.

The per-sample tolerances differ because the synthetic device, applied sample by sample at f_s, aliases the content it generates above Nyquist back into the band — exactly as a simulated device in the app does, and unlike a hardware device behind a converter's anti-alias filter. The aliasing is worst in the top half-octave of each order's band: over the full band, $\tanh(2x)$ departs from truth by up to 0.554 dB and the hard clip (a near-square wave whose spectrum falls only as $1/k^2$) by up to 3.1 dB. That this is the synthetic device and not the method is shown by each case's *phantom twin*: the same first nine Fourier amplitudes as an alias-free phantom, which the analyzer reads to within 0.01 dB over the whole band.

case	k	expected (dB)	S−P (dB)	S−T band (dB)	P−T band (dB)	S−T full (dB)
phantom	1	0	3.9e-15	0.0052	0.0052	0.0052
phantom	2	-10.458	5.3e-15	0.000851	0.000851	0.000851
phantom	3	-20	1.1e-14	0.000121	0.000121	0.000121
tanh	1	4.2083	4.4e-15	0.00492	0.00492	0.00492
tanh	3	-19.292	1.1e-14	0.000172	0.000172	0.000218
tanh	5	-40.883	1.4e-13	4.17e-05	4.17e-05	0.0446
tanh	7	-62.328	2.1e-12	4.29e-05	4.29e-05	0.299
tanh	9	-83.757	2.1e-11	8.3e-05	8.3e-05	0.554
tanh twin	1	4.2083	3.6e-15	0.00515	0.00515	0.00515
tanh twin	3	-19.292	1.4e-14	0.000169	0.000169	0.000169
tanh twin	5	-40.883	1.4e-13	7.21e-05	7.21e-05	7.21e-05
tanh twin	7	-62.328	1.6e-12	3.02e-05	3.02e-05	3.02e-05
tanh twin	9	-83.757	1e-11	7.73e-05	7.73e-05	7.73e-05
hardclip	1	-11.94	5.3e-15	0.0127	0.0127	0.0319
hardclip	3	-21.955	7.1e-15	0.0896	0.0896	0.548
hardclip	5	-27.372	1.4e-14	0.122	0.122	1.93
hardclip	7	-31.859	1.4e-14	0.202	0.202	2.01
hardclip	9	-36.348	1.4e-14	0.279	0.279	3.1
hardclip twin	1	-11.94	7.1e-15	0.00515	0.00515	0.00515
hardclip twin	3	-21.955	1.4e-14	0.000179	0.000179	0.000179
hardclip twin	5	-27.372	1.1e-14	5.82e-05	5.82e-05	5.82e-05
hardclip twin	7	-31.859	7.1e-15	2.04e-05	2.04e-05	2.04e-05
hardclip twin	9	-36.348	2.1e-14	6.05e-05	6.05e-05	6.05e-05
poly	1	0.47533	3.7e-15	0.00523	0.00523	0.00523
poly	2	-26.021	2.5e-14	0.000588	0.000588	0.000588
poly	3	-34.54	4.3e-14	0.000136	0.000136	0.000136
poly twin	1	0.47533	3.7e-15	0.00516	0.00516	0.00516
poly twin	2	-26.021	1.8e-14	0.00106	0.00106	0.00106
poly twin	3	-34.54	5e-14	9.32e-05	9.32e-05	9.32e-05

For $\tanh(2x)$ the exact amplitudes are $H_1 = 4.2083$ dB, $H_3 = -19.292$ dB and $H_9 = -83.757$ dB re input. A fifth case adds seeded white noise at -80 dBFS and four averaged repeats (the noise stream is reproduced exactly on both sides — same generator, same seeding). On the points the shipped predicate reads as shaped on both sides, the error against truth must sit within 3 times the read's own stated scatter; the last column is that ratio:

k	expected (dB)	shaped points	S−P (dB)	S−T shaped, band (dB)	error / allowance
1	4.2083	42	4.4e-15	0.00491	0.022
3	-19.292	42	1.1e-14	0.00246	0.036
5	-40.883	41	7.8e-14	0.0283	0.41
7	-62.328	38	8.7e-13	0.254	0.87
9	-83.757	36	1.2e-11	3.9	0.8

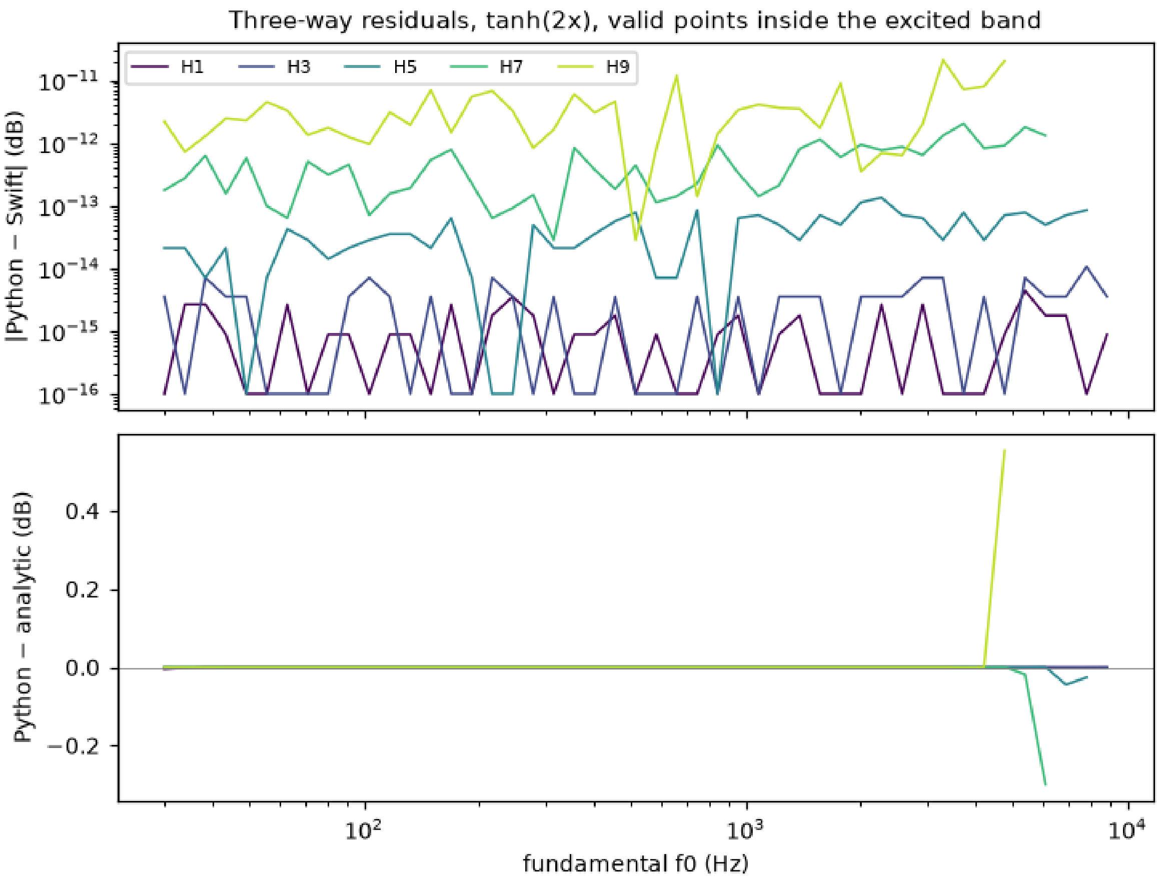

Figure 6: Three-way residuals for the worked example, every valid point inside the excited band. Top: |Python − Swift|, at the level of double-precision rounding. Bottom: Python minus analytic truth, showing the per-sample aliasing rising toward each order's band top.

3.8 Limits and failure modes

- **The fade edges are measurement, not device.** The last grid point sits on f_2, inside the fade-out, and reads the ramp (the phantom's H_1 there is 11.7 dB low); on a compensated record the calibrated-band bound removes it, and on a raw or simulated one the excited band is the honest limit. The first fraction of the sweep is treated the same way by every grid's $1.5f_1$ start.
- **Leakage between orders is bounded, and the bound is measured.** An order's read is floored by what the loudest order leaks into its window: worst at the band's low edge, and the value the presence predicate uses is the measured one, posted a hair below what the analyzer demonstrated.
- **A read is not automatically content.** The shaping predicate exists because a symmetric device's even orders are a read of its noise; every quantity computed from an unshaped read describes the noise.
- **Aliasing in a simulated device.** A device applied per sample at f_s folds its content above Nyquist into the band; the parity numbers above quantify it for two clippers. Hardware through a converter does not do this, and the alias-image bound (8) is about the *capture's* folding of the next order, a different mechanism.
- **Short sweeps carry fewer orders.** The 16-sample minimum half-width is a literal in the analyzer, pinned by the tool's guard rather than by the constants census; it binds only for sweeps far shorter than the standard plan.
- **The THD curve's top point.** The shipped curve requests 128 log-spaced points but its top point can land an ulp above f_2 and be dropped by the band guard; the manifest reports the delivered count.

4 Transfer Curve

4.1 Purpose, and what the measurement cannot tell you

The transfer curve draws a device's clipping shape directly: while a steady low-frequency sine plays, the instantaneous output is plotted against the instantaneous input over one averaged cycle. A memoryless device retraces a single curve, the static curve, and the figure's bends are its clipping; anything that makes the output depend on where the signal has just been — a shifting bias point, stored charge — opens the figure into a loop. The result is the averaged cycle itself (two vectors of n phase bins), the static curve, and three dimensionless loop areas: the total, the part a linear filter's phase rotation explains, and the part it does not.

What it cannot tell you, stated once: (a) anything about other drive levels — one curve is one operating point, and a clipper whose knee sits above the drive draws a straight line; (b) whether an open loop is memory or filter phase — ordinary tone filters rotate phase and open the loop without any memory, which is why the phase removal exists and why the raw figure's middle band of loop area is hedged; (c) the device's *level* asymmetry once a series coupling capacitor has removed the output's mean, which every pedal's output stage does (§4.8); (d) anything about a rig's own residual alignment, which draws a loop the measurement cannot distinguish from the device's (§4.8); and (e) anything about what the distortion sounds like.

4.2 The stimulus

The stimulus is a sine at the probe note f_0 (the standard note is E2, at the value the parameter table gives verbatim; other notes are exact twelve-tone equal temperament with A4 = 440 Hz), at a linear peak amplitude A:

$$x[n] = A \sin(2\pi f_0\, n/f_s), \qquad n = 0 \ldots \lfloor T f_s \rfloor - 1, \tag{11}$$

scaled over its first and last $\lfloor t_{\text{fade}} f_s \rfloor$ samples by the raised cosine $\frac{1}{2}(1 - \cos(\pi i/N_{\text{fade}}))$, and preceded and followed by silence (the pre-roll and the tail). The app plays this stimulus, measures the loop latency per capture, and drops the pre-roll before analysis; the oracle renders it through a synthetic device at zero latency. The standard amplitude is the fingerprint sweep's drive (−26 dBFS), so the curve and the other measurement kinds describe one operating point; the worked example below uses the loudest drive the hardware sheet offers, because at the standard drive the reference clippers draw a straight line, which is the point of the sheet's level control.

4.3 The pipeline

Given the tone $x[n]$ and the captured output $y[n]$, aligned by the integer loop latency ℓ:

1. **Phase binning.** With $P = f_s/f_0$ samples per cycle, every sample $i \geq \lfloor s\,P \rfloor$ (s the skipped opening cycles, while the circuit settles) with $i + \ell$ inside the capture is assigned to the bin

   $$b(i) = \min\Big(n - 1,\ \Big\lfloor n \cdot ((i/P) \bmod 1)\Big\rfloor\Big), \tag{12}$$

   and the averaged cycle is the per-bin mean of $x[i]$ and of $y[i + \ell]$ respectively: $\bar{x}_b$, $\bar{y}_b$, $b = 0 \ldots n - 1$. Noise averages; shape does not. Every sample to the end of the tone is binned, the fade-out included — only the start is skipped (§4.8).

2. **The box.** Every area is normalized by the rectangle that bounds the figure (box normalization):

   $$B = 4 \max_b |\bar{x}_b| \max_b |\bar{y}_b|. \tag{13}$$

3. **The total loop area.** The closed polyline $(\bar{x}_b, \bar{y}_b)$ is split at every proper self-crossing and the absolute shoelace areas of the resulting lobes are summed — the unsigned lobe area — so that opposite-winding lobes add instead of cancelling (a signed integral sees only the fundamental's quadrature and lets a figure-eight report zero). A lobe smaller than a fixed fraction of its own polyline's box is dropped as bin noise. The reported figure is total = lobes$(\bar{x}, \bar{y})/B$.

4. **The fundamental ellipse.** Both cycles are projected onto $\{1, \cos\theta_b, \sin\theta_b\}$, $\theta_b = 2\pi b/n$:

   $$a_x = \tfrac{2}{n}\sum_b \bar{x}_b \cos\theta_b, \quad b_x = \tfrac{2}{n}\sum_b \bar{x}_b \sin\theta_b, \quad a_y, b_y \text{ likewise}, \quad \bar{y} = \tfrac{1}{n}\sum_b \bar{y}_b, \tag{14}$$

   and the fundamental-only trajectory is the least-squares fundamental ellipse, whose area is analytic:

   $$\text{linear} = \frac{\pi\,|a_x b_y - a_y b_x|}{B}. \tag{15}$$

   For an input $A \sin\theta$ and an output whose fundamental is $R\sin(\theta + \varphi_1)$ this is $\pi A R\,|\sin\varphi_1|/B$ — a linear filter's rotation of the fundamental, never memory; for a linear device, whose peak is its fundamental's, it reduces to $(\pi/4)\,|\sin\varphi_1|$.

5. **The nonlinear residual.** The ellipse's output is subtracted and the lobes of what remains are summed:
$$\text{nonlinear} = \frac{\text{lobes}(\bar{x},\ \bar{y}_b - \bar{y} - a_y \cos\theta_b - b_y \sin\theta_b)}{B}, \tag{16}$$
the nonlinear loop area: the harmonic content's own lobes, the headline memory figure. A memoryless clipper's harmonics are in phase with the drive and retrace; harmonics rotated by a filter, or by genuine memory, enclose area here.

6. **The static curve.** The cycle is split at the bins of the input's minimum and maximum into a rising branch and a falling one, each reduced to a strictly ascending polyline in $\bar{x}$, interpolated linearly onto a common grid of inputs from $\min \bar{x}$ to $\max \bar{x}$, and the two interpolants are averaged — per-branch resampling, never binning by input value, whose sample density under a sinusoidal drive is $\propto 1/\sqrt{A^2 - x^2}$ and starves the centre.

7. **The memory verdict.** Taken on the nonlinear area (of the compensated figure when the removal is accepted, §4.4): below the first threshold the device reads memoryless-ish, above the second it reads memory, and between them the reading is hedged deliberately — tone filtering *ahead* of the clipper rotates every harmonic by k times the drive's rotation, and a single fundamental cannot separate that from mild memory. Both thresholds are literals in the shipped verdict; the manifest reads them by bisecting it.

Every derived quantity is recomputed from the stored cycle whenever a record is read, so a stored record carries the cycle and nothing else that a fix would have to migrate.

4.4 Read-time predicates on the result

Three judgements are made against the record's paired Harmonic Distortion analysis — the same device, the same setting, the swept measurement of §3 — when one exists; none is persisted.

Orientation. A plugin-path capture is aligned by correlation against the tone and can land half a cycle off, which reverses the figure and, for an asymmetric device, pairs $x(t)$ with $y(t + P/2)$. The orientation resolution reads the cycle's fundamental phase $\psi_1 = \angle Y[1] - \angle X[1]$ (direct DFTs of the two cycles); when it sits within the lock tolerance of $\{0, \pi\}$ the capture is detectably locked, and when the paired H_1's polarity confidently disagrees with $\cos\psi_1 < 0$, the output cycle is advanced by $n/2$ bins and the figure recomputed. Every other case displays the figure as captured and implies nothing. Polarity itself is the energy-weighted mean unit phasor of H_1 over the low band,
$$m = \frac{\sum_b |H_1[b]| \operatorname{Re} H_1[b]}{\sum_b |H_1[b]|^2}, \qquad \text{inverting} \Leftrightarrow m < 0, \quad \text{confidence} = |m|, \tag{17}$$
the real part and never the resultant length: a quadrature response is coherent yet polarity-mute.

Fundamental presence. The memory verdict, the polarity statement and the pairing verdict all key off a fundamental; the fundamental presence reading is taken from the paired record at f_0 (which separates orders far better than one averaged cycle) and from the cycle's own DFT only when the paired reading is undetermined. Where the fundamental is absent the page draws the raw figure and states none of the three.

The phase removal. Harmonics generated *behind* pre-clipper dispersion inherit $k\,\varphi_{\text{pre}}(f_0)$, while an H_1-based rotation would remove $\varphi_{\text{pre}}(kf_0)$; no single shift reconciles the two, so the removal is fitted against the paired record's own branch phases $\angle H_k(kf_0)$, which carry each harmonic's true burden $k\,\varphi_{\text{pre}}(f_0)+\varphi_{\text{post}}(kf_0)$ by construction. The steps:

1. **Fittable terms.** The cycle's harmonic phases are read from its DFT, input-relative: $\text{cyc}_k = \angle Y[k] - k\,\angle X[1]$. Order k votes when the paired record's validity bound accepts (k, f_0) and, for $k \geq 2$, both $|Y[k]|/|Y[1]|$ and $|H_k(kf_0)|/|H_1(f_0)|$ clear the magnitude floor; its weight is $w_k = \min(1, |Y[k]|/|Y[1]|)$ and its phase difference

$$d_k = \text{wrap}\big(\text{cyc}_k - \angle H_k(kf_0) - \gamma_k\big), \qquad \gamma_k = \text{wrap}\big((k-1)\,\pi/2\big), \tag{18}$$

   γ_k being the Novák factor between the cycle's convention and the deconvolver's. Fewer than the minimum number of terms means insufficient evidence, never a verdict.

2. **The frame fit.** The two captures each measured their own latency, so a shift between their frames is legitimate and must never be applied as rotation. One shift Δl (samples) is fitted by maximizing the weighted phase coherence

$$C(\Delta l) = \sum_k w_k \cos\big(d_k - k\,\omega_0\,\Delta l\big), \qquad \omega_0 = 2\pi f_0/f_s, \tag{19}$$

   by a coarse scan over $\pm P/2$ and a fine scan around its maximum (both steps are literals, stated in the parameter table): the frame fit. The residuals $r_k = \text{wrap}(d_k - k\omega_0\Delta l)$ give the pairing residual, the weighted RMS over $k \geq 2$ (the fundamental can always be absorbed into Δl, so consistency lives in the harmonics); above the tolerance the outcome is a pairing mismatch.

3. **The anchored sequential fold.** The frame-mapped branch phase $m_k = \text{cyc}_k - r_k$ is what the paired record predicts for harmonic k in the cycle's own frame. Its burden over the memoryless memoryless lattice $\{0, \pi\}$ is known only modulo π (the multiple removed is the device's own harmonic sign, which the figure keeps), and the folds must come out mutually consistent. The fundamental is anchored: its lattice point is π when the paired polarity reads inverting and 0 when not (the nearest point only when polarity is not confident, and then the sign is quotable only inside the lattice-sign confidence), giving $e_1 = \text{wrap}(m_1 - \sigma_1)$. Each higher voting order, in ascending order, folds against a trend: $k\,e_1$ for the first, and for every later one $a\,k + F(k)$, where $F(k) = \phi_1^u(k) - \phi_1^u(1) + (e_1 - \omega_0\Delta l)$ is the paired H_1's unwrapped phase *shape* anchored at the fundamental's burden (the post-clipper term) and the slope a is refit after each fold over the orders already folded (the pre-clipper term; the fundamental never votes in it):

$$\beta_k = m_k - \pi\,\text{round}\Big(\frac{m_k - \text{trend}_k}{\pi}\Big), \qquad a = \frac{\sum_{j\leq k} w_j\, j\,(\beta_j - F(j))}{\sum_{j\leq k} w_j\, j^2}. \tag{20}$$

   The per-order assumption is that the curvature beyond the ramp is under $\pi/2$ — true of the resistor–capacitor networks pedals are built from, and weakest at the lowest order, where it holds across every device class measured.

4. **The all-pass.** The correction is a smooth function of frequency, applied to every bin of the cycle's spectrum at unity magnitude: the deviations $\beta_k - ak - F(k)$ at the voting orders are interpolated by a shape-preserving cubic (Fritsch–Carlson, constant beyond the end nodes), and for $1 \leq k < n/2$

$$Y[k] \leftarrow Y[k]\,e^{-j\theta_k}, \qquad \theta_k = ak + F(k) + \text{dev}(k), \tag{21}$$

with DC and the real Nyquist bin untouched. The compensated cycle is the inverse real DFT and the figure is recomputed from it. Content above the voting orders survives phase-corrected rather than being resynthesized away, which is what removed the truncation ripple an earlier per-order resynthesis drew on hard-clipped plateaus. The correction is *measured* up to the support $k_{\max} f_0$, the highest voting order times the drive note; above it the ramp is extended, F follows H_1's phase as far as its grid holds a value, and the deviation holds its last knot — an extrapolation the page's caption states.

5. **The outcome.** One of five: compensated; a pairing mismatch (no shift reconciles the harmonic phases); insufficient evidence (too few fittable terms); a negligible raw loop (the total under the dead zone, decided after the fit ran so polarity still rides it, and outranking a mismatch — a noise-dominated tiny loop must not wear a warning); and a tripped metric guard, when a removal that passed the residual still grew a *decomposed* area beyond its tolerance (the unsigned total is never compared: the ellipse and the harmonic lobes partially cancel in it). A pairing mismatch has two possible causes the residual alone does not separate: a session mismatch — another setting or another time, resolved by re-measuring both kinds together — and a structural one, a device whose harmonics have no fundamental-anchored phase relationship (a frequency doubler), which no re-measurement resolves.

6. **Polarity, confirmed.** The figure states polarity only when a fit ran, its residual confirms the pairing, and the fundamental's lattice sign resolved; the robust H_1 statistic supplies the sign, the fit the trust. Absence is honest; a guessed sign is not.

An older correction — rotating harmonic k by $\angle H_1(kf_0)$ alone — remains in the kernel with no consumer, and is not the method this document describes.

4.5 Parameters

Every value below is read from the manifest, which read it from the shipped code; the two verdict thresholds were located by bisecting the shipped verdict, and the scan steps, the bin count, the grid, the polarity band and the analyzer's unused default are literals the manifest carries as rule strings.

Parameter	Value
Stimulus	
standard note / frequency f_0	E2 / 82.4 Hz
standard amplitude A (re full scale)	0.05 (-26.02 dBFS)
tone duration T / pre-roll / tail	1.5 s / 0.2 s / 0.2 s
fade-in and fade-out t_{fade}	0.02 s
skipped cycles (the app's call)	8 (the analyzer's own default is 5, unused)
sample rate f_s (the manifest's)	96 000 Hz
samples per cycle P at f_0	1165.0485
probe notes offered	E2, A2, E3, A3, E4, A4, E5, A5, A6, C7
note-range capture	E2, A3, A4
drive level bounds (hardware / plugin ceiling)	-60 to -6 dBFS / 0 dBFS
Binning and the figure	
phase bins n	256
static-curve grid points	101
lobe floor (fraction of the box)	0.0002
Memory verdict (bisected from the shipped verdict)	
memoryless-ish below	0.05
memory effects from	0.15

Parameter	Value
Phase removal (§ 6.5)	
dead zone ε (raw total area)	0.01
pairing residual tolerance	0.25 rad
fit magnitude floor (both sides)	0.005 (-46 dB)
minimum fittable terms	3
lattice sign confidence	0.6 rad
metric-guard tolerance (decomposed areas)	0.02
frame-fit scan: coarse / fine step	0.05 / 0.001 samples (literals)
Orientation and polarity	
half-cycle lock tolerance	0.05 rad
polarity confidence threshold	0.35
polarity band	20 Hz – 2 kHz (literals)
Fundamental presence	
analyzer separation bound	76 dB
Oracle (`analysisdump transfer-synth`)	
biquad default Q	0.70710678
truth series orders K	200
truth quadrature points	262 144

4.6 The result

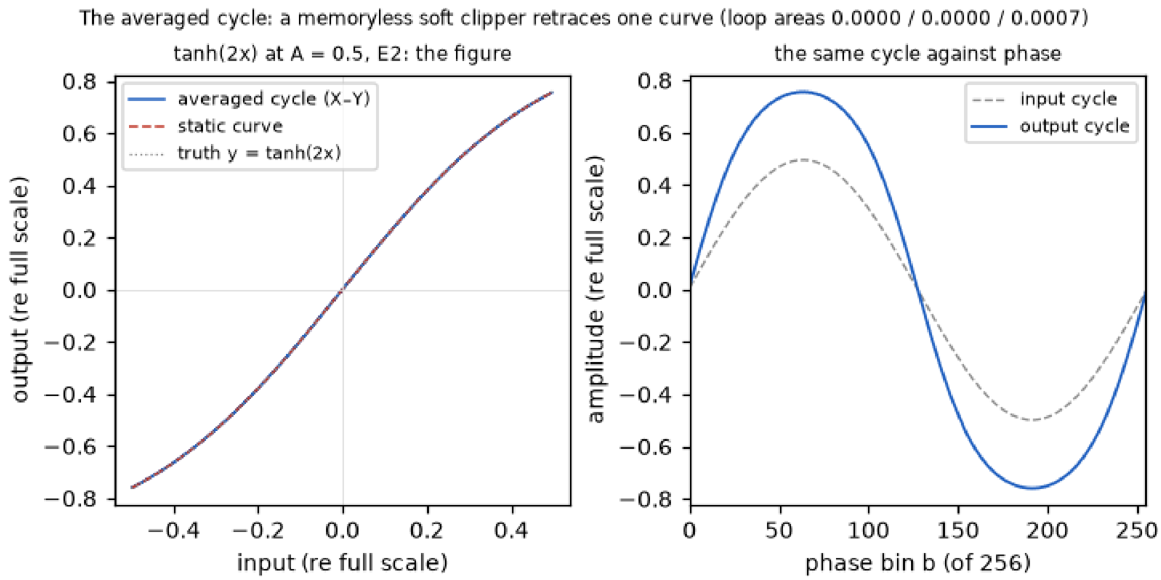

Figure 7: The worked example, $y = \tanh(2x)$ at $A = 0.5$ and E2, no filter: the averaged cycle as the X–Y figure with the static curve and the exact curve overlaid (left; the three coincide), and the same cycle against phase (right). Every loop area is zero to the bin floor.

The app's own rendering of the same measurement kind, from the user guide's figure pipeline (a simulated device, rendered headlessly from the chart view with the removal on):

4.7 Worked example and parity

The parity test runs 21 synthetic cases through both implementations, each a known device — one of the reference nonlinearities between an optional pre-filter and an optional post-filter, each

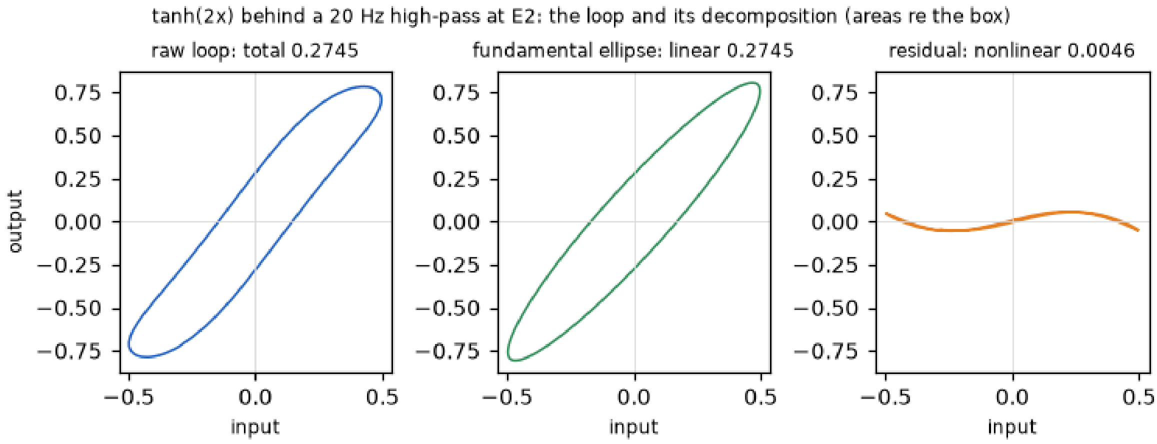

Figure 8: The same clipper behind a 20 Hz second-order high-pass at E2: the raw loop, the fitted fundamental ellipse and the residual after the ellipse's output is subtracted, each with its normalized area. The filter draws the ellipse; the residual holds only the harmonics' own rotation.

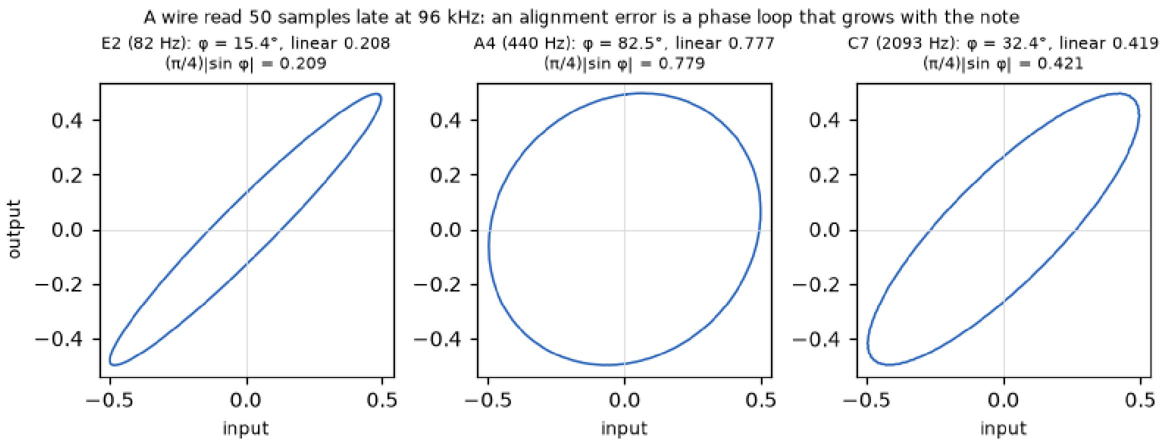

Figure 9: A wire, analyzed 50 samples late at 96 kHz, at three notes: an alignment error is a phase loop that grows with the note. Each title gives the phase the error implies at that note, the measured linear area and its closed form $(\pi/4)|\sin\varphi|$.

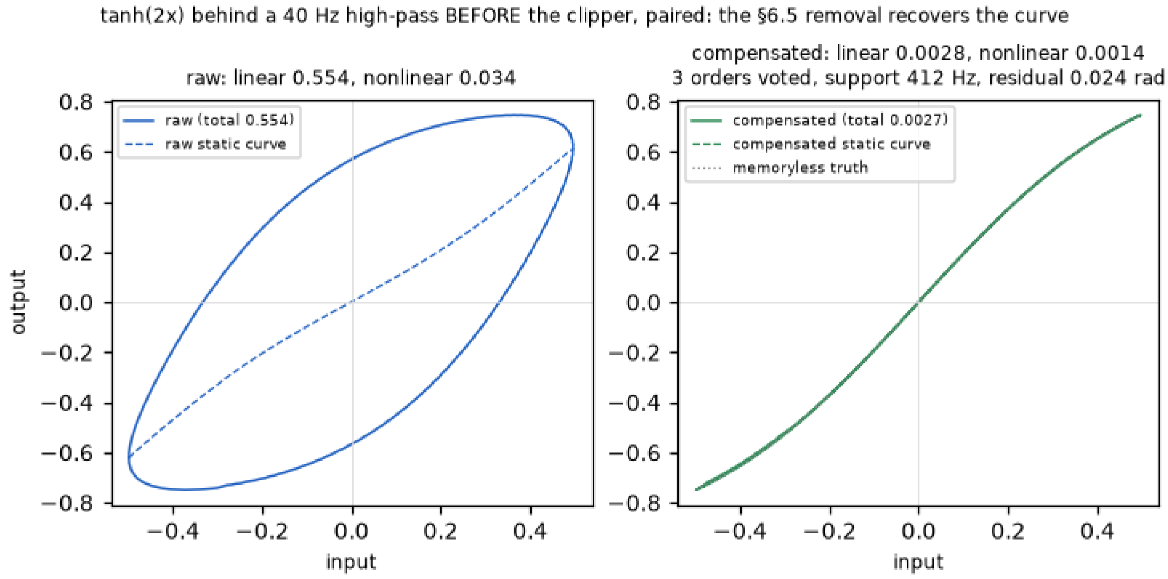

Figure 10: The soft clipper behind a 40 Hz high-pass placed *before* the clipper — the pre-dispersive case an H_1-only rotation cannot handle — paired with its own swept analysis. Left: the raw figure and its static curve. Right: the compensated figure over the memoryless truth, with the orders that voted, the support and the residual.

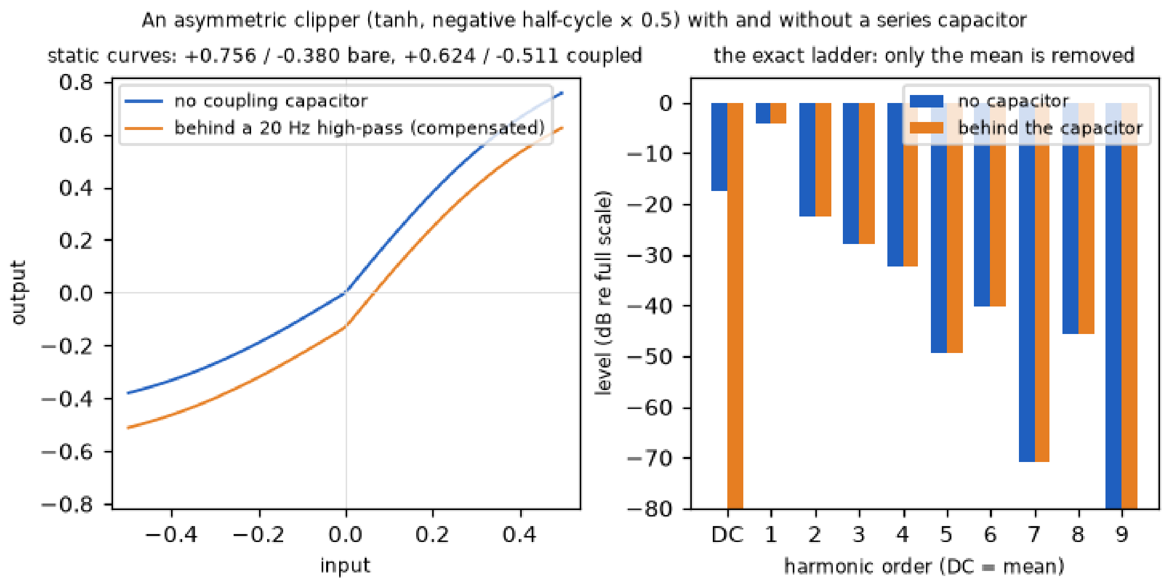

Figure 11: An asymmetric clipper (the negative half-cycle scaled by one half) with and without a series capacitor: the static curve's unequal clip levels are centred by the capacitor while the exact harmonic ladder loses only its mean — the even harmonics survive.

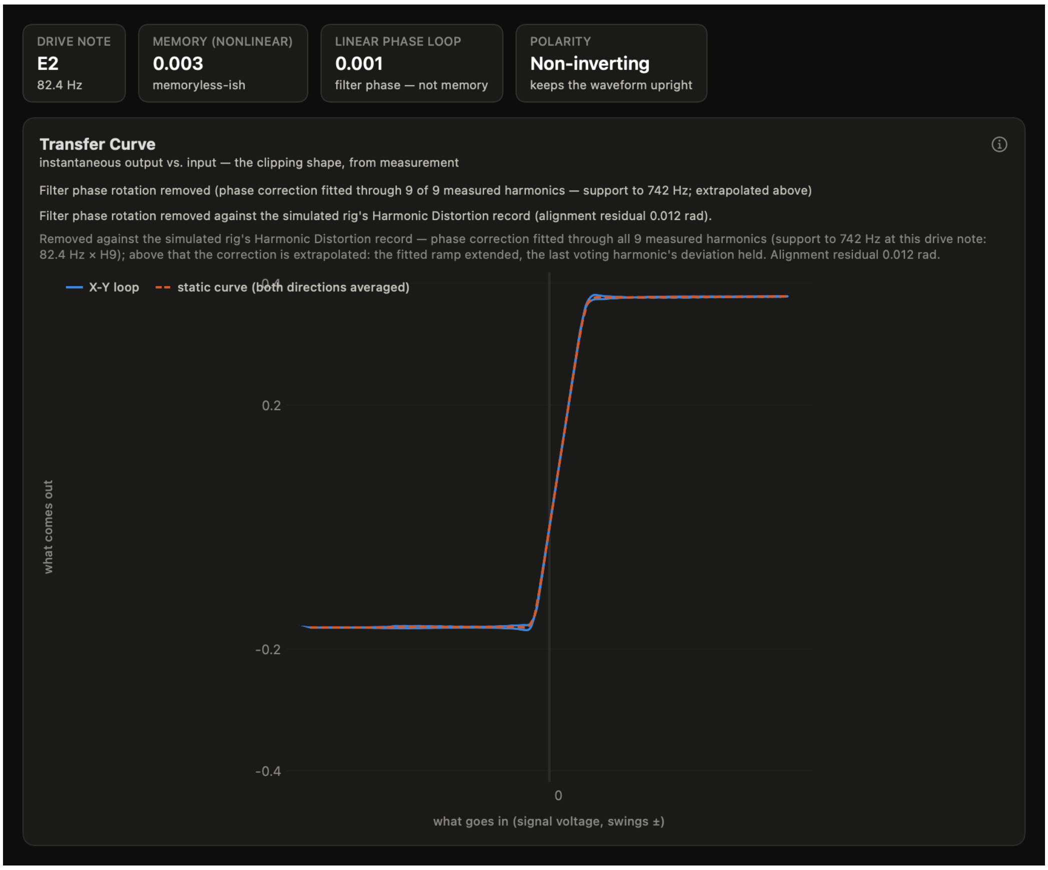

Figure 12: The Transfer Curve view as the app draws it, from the guide's simulated fixture, with the phase removal applied and its caption stating the support and the residual.

a second-order (Butterworth) low-pass or high-pass — driven by the plan's own tone and, where paired, measured by the swept method of §3 for its reference. Beside every measured quantity the oracle prints a truth computed by quadrature: for a memoryless device with no filter the curve $y(x)$ itself; otherwise the Fourier series of the clipper's output at the pre-filter's delivered drive, each order scaled and rotated by the post-filter's analytic response and by any planted alignment error, and — for the compensated figure — the same series with every phase returned to the lattice and every magnitude kept. The test asserts:

- Swift against Python: every cycle value, static-curve value, compensated value, area, ψ_1, residual and fit term within 1e-10 (measured maximum 7.4e-13 on this run), the frame shift within 1e-09 samples (measured 0), and every discrete reading — class, flip, polarity, tile, verdict, presence — identical;
- the fundamental ellipse against its closed form $(\pi/4)\,|\sin\varphi_1|$ on a linear device: a ratio within 0.01 of one at the plan's tone (measured deficit at worst 0.73 %) and within 0.001 at ten times its length (measured 0.05 %);
- the raw static curve against the truth, as a fraction of the output peak: 0.01 for smooth memoryless devices (measured 0.003 on the soft clipper, 4.4e-05 at the standard drive), 0.05 for the hard clip (measured 0.027: a 256-bin cycle rounds a corner by about an eighth of a bin's input span), 0.05 for an open ellipse (the branch-mean curve of an open loop is least trustworthy where the branches meet); and a memoryless device's nonlinear area under 0.005 (measured 0.002);
- every paired filtered device compensates, finds the planted frame shift within 0.5 samples (measured 0.39), keeps its residual under 0.05 rad (measured 0.0237), recovers the memoryless truth to 0.01 of peak on the static curve (measured 0.0055) and 0.02 on the cycle, and reads both compensated areas under 0.005 (measured 0.0028).

The removal's parity is established: every paired filtered case compensates on both sides and the two implementations agree on the compensated figure to the Swift-vs-Python tolerance.

The memoryless cases, at the loudest hardware drive and (one case) the standard drive, where the soft clipper is within its linear region:

case	A	total	linear	nonlinear	static vs truth (of peak)
a-tanh	0.5	0	5.95e-06	0.000719	0.00298
a-tanh-plan	0.05	0	4.52e-08	0.000612	4.41e-05
a-hardclip	0.5	0.00132	9.68e-05	0.00196	0.0266
a-asym	0.5	0	0.000119	0.000458	0.00337
a-param	0.5	0	1.41e-05	0.000939	0.00747

The linear devices — the identity behind the low-pass at three notes and under a 50-sample alignment error at three notes — at the plan's tone and at ten times its length:

case	f_0 (Hz)	φ_1 (°)	measured	$(\pi/4)\lvert\sin\varphi_1\rvert$	ratio
b-lp-E2	82.4	-4.3	0.0586	0.058997	0.99327
b-lp-A6	1 760	-100.2	0.77136	0.7729	0.998
b-lp-C7	2093	-113.4	0.71634	0.72094	0.99362
c-delay-E2	82.4	15.5	0.20771	0.20923	0.99275
c-delay-A4	440	82.5	0.77706	0.77868	0.99792
c-delay-C7	2093	32.4	0.41926	0.42128	0.9952
b-lp-E2-15s	82.4	-4.3	0.058976	0.058997	0.99965
b-lp-A6-15s	1 760	-100.2	0.77283	0.7729	0.9999
b-lp-C7-15s	2093	-113.4	0.72058	0.72094	0.9995

The paired cases: two post-filters and the pre-filter on the soft clipper, the asymmetric clipper behind a DC-blocking high-pass, an inverted device, the half-cycle lock and the unfiltered clipper (whose raw loop is under the dead zone, so the removal is a no-op and the fit still confirms polarity):

case	class	voted	support (Hz)	residual (rad)	Δl (samples)	comp. lin.	comp. nonlin.	comp. vs truth (of peak)
d-tanh-lp	compensated	3	412	0.00322	-0.172	0.00049	0.00054	0.0044
d-tanh-hp	compensated	3	412	0.00945	-0.388	0.00024	0.00066	0.0022
e-tanh-prehp	compensated	3	412	0.0237	-0.272	0.0028	0.0014	0.004
f-asym-hp	compensated	7	659.2	0.0108	-0.102	0.0015	0.0008	0.0055
g-tanh-inv-lp	compensated	3	412	0.00322	-0.172	0.00049	0.00054	0.0044
g-tanh-shift	negligible loop	3	412	0.00833	0.401	–	–	–
g-tanh-paired	negligible loop	3	412	0.00816	-0.0763	–	–	–

Two readings from those tables are worth stating in words. The pre-dispersive case's raw nonlinear area, 0.0343, sits in the hedged band of the verdict with no memory in the device at all, and the removal takes it to 0.0014 — the mid-band hedge in the pipeline is exactly this case. And the half-cycle lock is undone: the captured ψ_1 read 3.144 rad, 0.0026 from the lattice, the paired polarity read non-inverting, and the resolved figure ascends.

4.8 Limits and failure modes

- **The normalization is the box's, and the constant is $\pi/4$ only for a linear device.** The ellipse area is $\pi AR|\sin\varphi_1|/(4A\,y_{\max})$; on a clipped output the peak is not the fundamental's and the figure is $(\pi/4)(R/y_{\max})|\sin\varphi_1|$, which can exceed $(\pi/4)|\sin\varphi_1|$ (a square wave's fundamental is $4/\pi$ of its peak). A reader converting a linear-device figure to an angle uses $\varphi_1 = \arcsin(4\,\text{linear}/\pi)$. On the synthetic single-pole check the shipped code reads the closed form to 0.73 % at the plan's tone and 0.05 % at ten times its length, so a deficit of several percent on a hardware capture is in the measurement chain, not in the metric's constant.

- **The fade-out is binned.** The skipped opening cycles protect the start; every sample to the end of the tone is binned, the last t_{fade} under its ramp, a phase-dependent weighting whose share of the average falls with the tone's length. It is the whole of the deficit above (the E2 ratio moves from 0.9933 to 0.99965 between the two lengths), and it lowers the captured input peak by a few tenths of a percent at the plan's length.

- **A residual alignment error is a loop.** An residual alignment error of e samples rotates the fundamental by $\varphi = 2\pi f_0 e/f_s$ and draws a linear area of $(\pi/4)|\sin\varphi|$ on a wire — growing with the note (Figure 9). The integer latency the app removes leaves at least the fractional part, and a rig can leave more; on a device, that loop is indistinguishable from the device's own linear-phase loop. Two notes measured on one rig let a reader test the hypothesis: a single e must reproduce both deficits. The nonlinear area is untouched by it on a linear device.

- **The coupling capacitor removes level asymmetry, not harmonic asymmetry.** A clipper clamped at $+a$ and $-b$ at roughly equal duty carries a mean near $(a-b)/2$; a series capacitor at the output strips it, leaving levels of $\pm(a+b)/2$ — equal magnitudes from an unequal circuit (Figure 11). The shape asymmetry remains, and the even harmonics in the paired swept analysis are where the asymmetry is read. Because every pedal has one, a transfer curve essentially never shows the raw asymmetry of an asymmetric clipper.

- **The support ends at the highest voting order.** A harmonic too quiet to clear the magnitude floor on either side does not vote (the soft clipper at this drive seats 3 orders, the asymmetric clipper 7); above the support the correction is the extended ramp, the H_1 shape and a held deviation, and a hard-clipped device has content far above it.

- **The branch phase is read at the nearest bin.** The paired record's phase at kf_0 is the nearest bin's, so a read can be off by up to half a bin width times the filter's phase slope, which the frame fit absorbs as a fictitious shift of a fraction of a sample (measured up to 0.39 samples on the parity cases) and leaves in the higher orders' residuals.
- **The bins round a corner.** With n bins per cycle a hard clip's corner is averaged over a bin's input span, and the static curve departs from the exact curve there by a few percent of the clip level; the smooth devices read under one percent of peak.
- **The static curve of an open loop.** The branch-mean estimate is least trustworthy where the two branches meet, at the input's extremes; on a wide ellipse it departs from the true line by a few percent of peak there. The app draws it dashed on an open loop for that reason.
- **A declined removal names its class, not its cause.** The residual separates a genuine pairing from a mismatch; it does not separate a session mismatch from a structural one, and the two need different remedies (§4.4).

5 Glossary

Generated from `Docs/Methods/terms.yaml`; the plain-language companion prints the same entries in plain language.

alias image Content above half the sample rate folds back below it in a sampled capture; order $k+1$'s image collides with the read of order k once $f_0 > f_s/(2k+1)$.

averaged cycle The stimulus and the capture folded onto one period of the drive note in n equal phase bins, each bin holding the mean of every sample whose phase fell in it, from the skipped opening cycles to the end of the tone; the figure's two coordinates are the two bins' means.

box normalization Every loop area is divided by $4\,x_{\max}\,y_{\max}$, the rectangle that bounds the figure, so the areas are dimensionless and a pure ellipse from a phase shift φ reads $(\pi/4)\,|\sin\varphi|$.

clipping A memoryless nonlinearity that limits the output amplitude: the output follows the input up to a threshold and is compressed or held beyond it (hard: a corner into a flat ceiling; soft: a gradual bend). An odd-symmetric clipper produces odd harmonics only; any asymmetry between the two half-cycles adds even harmonics.

coherence (Γ) $|\sum_i z_i|/\sum_i |z_i|$ over lagged products $z_i = H(b_i + \mathrm{lag})\,\overline{H(b_i)}$: shaped content is a phasor whose phase advances smoothly with frequency, noise scatters isotropically, and the statistic is $1/\sqrt{n}$ for pure noise.

coupling capacitor A series capacitor at a device's output: a first-order high-pass whose corner sits far below the probe note, so it removes the output's mean and leaves every harmonic's magnitude essentially untouched — an asymmetric clipper's unequal clip levels are centred by it while its even harmonics survive.

dBc Decibels relative to a carrier — here the fundamental, or the louder of two played tones — so that a product's level is stated as a ratio to the signal that produced it.

dBFS Decibels relative to digital full scale: $20\log_{10}$ of a linear amplitude in which 1.0 is the largest value the converter or file can represent.

decibel (dB) A logarithmic ratio: $20\log_{10}$ of an amplitude ratio (equivalently $10\log_{10}$ of a power ratio). $+6\,\mathrm{dB}$ doubles an amplitude, $-20\,\mathrm{dB}$ is one tenth, $-40\,\mathrm{dB}$ one hundredth; the reference is named by a suffix or by context.

deconvolution Dividing the captured response's spectrum by the sweep's spectrum — here, multiplying by the analytic inverse filter — to recover the device's impulse response, in which the harmonic pulses appear at their time advances.

derotation Multiplying every harmonic response by $e^{+j2\pi f\delta/f_s}$ on its own output-frequency axis to remove a capture delay δ — a unity-magnitude rotation, so magnitudes are untouched at every bin.

excited band The range of fundamentals the sweep drove at full amplitude: from the instantaneous frequency where the fade-in ends to the one where the fade-out begins. Outside it the device was driven under a ramp and a read describes the measurement, not the device.

extraction window (T_k) The raised-cosine window that cuts order k's pulse out of the impulse response; its half-widths are half the gap to each neighbouring pulse, capped, and its duration sets the order's spectral resolution.

frame fit The single time shift Δl between the transfer capture's alignment and the paired record's, found by maximizing the magnitude-weighted phase coherence of every fittable harmonic; its residual over the harmonics is the pairing statistic.

fundamental (f_0) The excitation frequency whose harmonics are being read; in a swept measurement every f_0 in the band is excited in turn, and the k-th harmonic of f_0 is read at the output frequency kf_0.

fundamental ellipse The least-squares ellipse through the trajectory: the projection of both cycles onto the fundamental. Its area is the linear-phase loop — what a filter's rotation of the fundamental contributes, never memory.

fundamental presence Whether $H_1(f_0)$ is a measurement at all: absent when it sits at or beyond the analyzer's measured separation bound below the loudest valid in-band order, or at or below its rig-floor margin. Where absent, no ratio to it is quoted.

harmonic order (k) The integer multiple of the fundamental at which a harmonic sits; order 1 is the fundamental itself. Odd orders are produced by symmetric nonlinearities, even orders require asymmetry.

harmonic pulse In the deconvolved response, the impulse response belonging to one harmonic order, arriving $L \ln k$ seconds before the linear one; each is cut out with its own window.

harmonic response ($H_k(f)$) The complex, input-normalized amplitude with which the device emits energy at output frequency f as its k-th harmonic; the harmonic of a fundamental f_0 is read at $f = kf_0$.

harmonic shaping The read-time predicate deciding whether a harmonic read is content the circuit deterministically shaped or noise that landed in that order's window, from the coherence of the complex response along frequency.

impulse response ($h[n]$) The deconvolved response, circular over the analysis length N: the linear pulse at index 0 and order k's pulse at $N - \mathrm{round}(L \ln k \cdot f_s)$.

input-normalized Divided by the sweep amplitude A, so that a unit-gain linear device reads $H_1 = 1$ (0 dB) and every H_k is a ratio to the input, not an absolute level.

instantaneous frequency ($f(t)$) The frequency the sweep is passing through at time t: $f(t) = f_1 e^{t/L}$, the derivative of the phase divided by 2π.

inverse filter ($\tilde{X}^{-1}(f)$) The closed-form stationary-phase inverse of the synchronized sweep, $2\sqrt{f/L}\, \exp(j(\pi/4 - 2\pi f L(1 - \ln(f/f_1))))$, tapered to zero below f_1; using the closed form extrapolates the phase law above f_2, so harmonics above the sweep's top deconvolve cleanly.

measurement floor The level below which a read is the rig rather than the device, derived at read time from the embedded calibration's pooled distortion stamp with per-track, drive and averaging adjustments; a read at or below it is reported as a bound.

memory verdict The three-way reading taken on the nonlinear loop area (of the compensated figure when the phase removal is accepted): memoryless-ish below the first threshold, memory effects at and above the second, and a deliberately hedged band between them, where tone filtering ahead of the clipper rotates every harmonic by k times the drive's rotation and a single fundamental cannot separate that from mild memory. Both thresholds are literals in the shipped verdict, which the manifest reads by bisection.

memoryless lattice The set of harmonic phases $\{0, \pi\}$ (in the cycle's convention, after the Novák factor $(k-1)\pi/2$) at which a memoryless device's harmonics sit; each measured harmonic's excess over its nearest lattice point, folded consistently from low order to high, is the filter burden the removal takes out.

methods manifest A machine-readable JSON file, emitted by the shipped measurement kernel through `analysisdump manifest`, listing every parameter that determines a measurement's result — each value obtained by calling the function that owns it. The documents' tables are generated from it and CI fails when the committed copy drifts from a fresh one.

nonlinear loop area The unsigned lobe area of the residual trajectory after the fundamental ellipse's output is subtracted — the harmonic content's own lobes, the headline memory figure the verdict is taken on.

orientation resolution A plugin-path capture is aligned by correlation against a tone and may land half a cycle off; when the cycle's fundamental phase sits on the $\{0, \pi\}$ lattice and the paired polarity confidently disagrees, the output cycle is advanced half a period, restoring the true input–output pairing.

pairing residual The magnitude-weighted RMS phase residual, in radians, over the harmonics of order two and above after the frame fit; a genuine pairing sits well under the accept bar and a record of another session or setting, or a device whose harmonics have no fundamental-anchored phase relationship, does not.

parity test A continuous-integration test that runs the shipped code and the independent Python reimplementation on identical synthetic devices and asserts, at every valid point, agreement between the two to a stated tolerance and agreement of both with analytic ground truth to a stated tolerance.

phantom capture A synthetic capture $\sum_k m_k A \sin(k\varphi(t))$ built from the sweep's own phase law, so that its k-th harmonic response is exactly m_k by construction, with no content above the listed orders and no aliasing; the oracle's exact case.

phase removal The § 6.5 correction: the capture's harmonic phases are fitted against the paired Harmonic Distortion record's branch phases with one frame shift, each harmonic's burden over the memoryless lattice is folded out sequentially, and the correction is applied as a full-band unity-magnitude all-pass, so filter phase before and after the clipper is removed and only unpredicted phase — memory evidence — remains.

polarity Whether the device inverts: read from the paired record's H_1 as the energy-weighted mean unit phasor over the low band, whose real part is the confidence (a quadrature response is coherent yet polarity-mute); stated on the figure only when the frame fit confirms the pairing.

rate constant (L) The time in seconds over which the sweep's instantaneous frequency grows by a factor of e: $f(t) = f_1 e^{t/L}$. Fixed by the synchronization condition from the requested duration.

read scatter The one-sigma uncertainty, in dB, of a magnitude read at its estimated signal-to-noise ratio — the Rician mean-of-dB statistic, pinned by a Monte-Carlo table and saturating at the pure-noise value.

residual alignment error Any difference between the integer loop latency removed before binning and the true delay, in samples; it rotates the output's fundamental by $2\pi f_0 e/f_s$ and harmonic k by k times that, opening the figure into a phase loop that grows with the note.

static curve The output-versus-input estimate that ignores the loop: the cycle is split at the input's extremes into a rising and a falling branch, each is interpolated onto a common input grid, and the two are averaged — never binned by input value.

sweep averaging Combining N back-to-back sweeps from one capture by complex, bin-wise averaging of their harmonic responses after each repeat is derotated by its fitted capture delay; the noise power falls as $1/N$ where the residual is noise, and magnitudes are never averaged.

synchronization condition The requirement that $f_1 L$ be an integer, met by rounding the rate constant; it makes the sweep's phase at every harmonic's arrival a whole number of cycles, so harmonic responses separate with meaningful phase.

synchronized swept sine ($x(t)$) A sine whose frequency rises exponentially from f_1 to f_2 over the duration T, with its rate constant chosen so that $f_1 L$ is an integer; under that condition each harmonic a memoryless device generates is an exact time-advanced copy of the sweep, advanced by $L \ln k$.

tanh The hyperbolic tangent, as the saturating function $y = \tanh(gx)$: linear for small inputs and bending smoothly toward ± 1, which it never reaches — the textbook soft clipper. It is odd-symmetric, so it produces odd harmonics only, and its ladder falls off steeply with order. The worked example throughout these documents is $\tanh(2x)$ at a sweep amplitude of 0.5, whose exact harmonic amplitudes are computed by quadrature.

total harmonic distortion (THD) $\sqrt{\sum_{k\geq 2} |H_k(kf_0)|^2} / |H_1(f_0)|$ over the orders that are in the audible band and pass the validity bound — THD, not THD+N; undefined where the fundamental is not present.

transfer curve The X–Y figure of a device under a steady low-frequency sine: instantaneous output against instantaneous input over one phase-averaged cycle. A memoryless device retraces a single curve; linear filter phase and genuine memory open it into a loop.

unsigned lobe area The total loop area: the closed trajectory is split at every self-crossing and the absolute area of every lobe is summed, so opposite-winding lobes add instead of cancelling; lobes under a floor fraction of the box are dropped as bin noise.

validity bound Where a harmonic read is trustworthy: $f_1 \leq f_0 \leq f_2$ and $f_0 \leq f_s/(2k+1)$ (the alias-image bound), plus $f_0 \leq f_{\text{cal}}/k$ on a compensated record. Reads outside it are excluded from every fit, feature, summary and comparison.

A Scripts, tests and how to reproduce

Paths are relative to `Docs/Methods/` in the repository, except the tool, which lives under `Tools/`. The repository: https://github.com/PhaseDog/PedalScope

Path	Role
`Tools/analysisdump`	the shipped-code oracle: `synth` (a known device through the shipped analyzer, with exact truth beside it), `transfer-synth` (a known device through the shipped transfer path, paired, with its truth beside it), `windows`, `manifest`
`generated/manifest.json`	the manifest, committed, drift-checked
`py/harmonic_distortion.py`	the reimplementation; `-help` lists the same options as `synth`
`py/parity_hd.py`	the comparison and the tolerances, with their provenance
`py/test_parity_hd.py`	the CI parity test (pytest)
`py/transfer_curve.py`	the Transfer Curve reimplementation (the options of `transfer-synth`)
`py/parity_transfer.py`	its comparison and tolerances, with provenance
`py/test_parity_transfer.py`	its CI parity test
`py/make_figures.py`	the figures, seeded, byte-stable, with TSV sidecars of the plotted values
`py/gen_docs.py`	the generated fragments this document inputs
`terms.yaml`	the terms source; `py/test_terms.py` is its guard
`make methods-test`	parity and terms guards
`make methods-docs`	manifest → figures → fragments → both PDFs

Toolchain: Python 3 with the versions pinned in `Docs/Methods/requirements.txt`; LaTeX through `latexmk` with `pdflatex`, pinned in `Docs/Methods/tex/latexmkrc`. The Swift tool builds with the repository's own toolchain.

References

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